

## DAY TWENTY THREE

# Magnetism

### Learning & Revision for the Day

- Magnetic Dipole Moment
- Magnetisation of Materials
- Electromagnet
- Magnetic Field Lines
- Magnetic Materials
- Permanent Magnet
- The Earth's Magnetism
- Hysteresis Curve

A naturally occurring ore of iron, magnet attracts small piece of iron towards it. The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore is called magnetism.

## Magnetic Dipole Moment

The simplest magnetic structure is the magnetic dipole characterised by a magnetic dipole moment  $\mathbf{M}$ .

A bar magnet exhibits two important properties, namely

- (i) the attractive property                      (ii) the directive property

If  $m$  is the pole strength and  $2l$  is the magnetic length of the bar magnet, then its magnetic moment is

$$\mathbf{M} = m(2l)$$

- Magnetic moment is a vector whose direction is from S-pole towards N-pole.
- The magnetic moment produced due to motion of electron,

$$M = iA = -\frac{evr}{2} = -n\left(\frac{eh}{2\pi m}\right) \quad \left[\because mvr = \frac{nh}{2\pi}\right]$$

## Magnetic Field due to a Bar Magnet

The magnetic field in free space, at a point having distance  $r$  from the given bar magnet (or magnetic dipole) is calculated in two conditions, along **axial line** and along **equatorial line**.

- **Along axial line**  $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}$

and the direction of  $\mathbf{B}$  is the same as the direction of  $\mathbf{M}$ . For a short dipole (or for a far away point on the axis) when  $r \gg l$ , the above relation is simplified as

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{r^3}$$



- **Along the equatorial line**

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{(r^2 + l^2)^{3/2}}$$

If  $r \gg l$ , the relation is modified as,  $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3}$

However, along the equatorial line, the direction of  $\mathbf{B}$  is opposite to that of  $\mathbf{M}$ .

In general, in a direction making an angle  $\theta$  with the magnetic axis, the magnetic field is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3} \sqrt{(3 \cos^2 \theta + 1)}$$

In these relations,  $\mu_0$  is a constant having a value of  $4\pi \times 10^{-7} \text{ T mA}^{-1}$  and it is known as the **magnetic permeability of free space**.

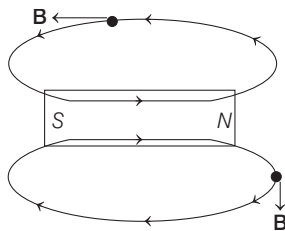
For solenoid  $\mathbf{B} = \mu_0 ni$

where,  $n$  is number of turns per unit length of solenoid and  $i$  the current through it.

## Magnetic Field Lines

The magnetic field lines is defined as the path along which the compass needles are aligned. They are used to represent magnetic field in a region.

- Magnetic field lines are closed continuous curves.
- Tangent drawn at any point on magnetic field lines gives the direction of magnetic field at that point.
- Two magnetic field lines cannot intersect each other.
- Outside a magnet, they are directed from north to south pole and inside a magnet they are directed from south to north.



## Torque on a Magnetic Dipole in a Magnetic Field

A magnetic dipole when placed in an uniform magnetic field, does not experience any net force. However, it experiences a torque given by

$$\tau = \mathbf{M} \times \mathbf{B} \text{ or } \tau = MB \sin \theta$$

where,  $\theta$  is the angle from the magnetic field, along which the dipole has been placed.

- **Work done** in rotating a magnetic dipole in a uniform magnetic field from an initial orientation  $\theta_1$  to the final orientation  $\theta_2$ , is given by

$$W = MB(\cos \theta_1 - \cos \theta_2).$$

- **Potential energy** of a magnetic dipole placed in a uniform magnetic field, is given by  $U_B = -\mathbf{M} \cdot \mathbf{B} = -MB \cos \theta$

- **The magnetic compass** (needle) of magnetic moment  $M$  and moment of inertia  $I$  oscillate in the magnetic field  $B$ . Then, its time-period is

$$T = 2\pi \sqrt{I / MB}$$

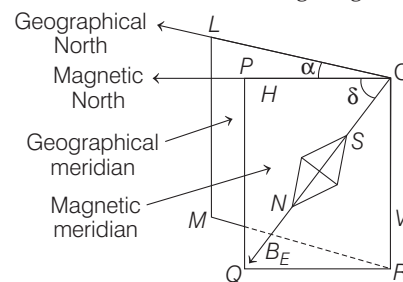
- Behaviour of a magnetic dipole in a magnetic field, is similar to the behaviour of an electric dipole in an electric field. However, the constant  $\frac{1}{4\pi\epsilon_0}$  is replaced by  $\frac{\mu_0}{4\pi}$ .
- If a magnetic dipole is in the form of a wire or a thin rod, when bent, its magnetic dipole moment  $\mathbf{M}$  changes because the separation between its poles has changed.

## The Earth's Magnetism

The earth is a natural source of magnetic field, a magnetic field is always present everywhere near the surface of the earth.

A freely suspended magnet always points in the north-south direction even in the absence of any other magnet. This suggests that the earth itself behaves as a magnet which causes a freely suspended magnet (or magnetic needle) to point always in a particular direction : north and south. The shape of earth's magnetic field resembles that of a bar magnet of length one-fifth of earth's diameter buried at its centre.

Magnetic field of earth is shown in the figure given below.



## Magnetic Elements of Earth

- **Angle of Declination ( $\alpha$ )** At a given place, the acute angle between the magnetic meridian and the geographical meridian is called the angle of declination (or magnetic declination)  $\alpha$  at that place.
- **Angle of Inclination or Dip ( $\delta$ )** The angle of dip  $\delta$  at a place is the angle which the direction of the earth's total magnetic field  $B_E$  subtends with the horizontal direction.
- **Horizontal Component of the Earth's Magnetic Field ( $B_H$ )** As earth's magnetic field, in general, is inclined at an angle  $\delta$  with the horizontal direction, it may be resolved into horizontal component  $B_H$  and a vertical component  $B_V$ ,

where  $B_H = B_E \cos \delta$  and  $B_V = B_E \sin \delta$

$$\Rightarrow B_E = \sqrt{B_H^2 + B_V^2}$$

$$\text{and } \tan \delta = \frac{B_V}{B_H}$$

## Magnetisation of Materials

There are some substances/materials which acquire magnetic properties on placing them in magnetic field, the phenomena is called magnetisation of materials. To describe the magnetic properties of material, we have to understand the following terms:

(i) **Magnetic Induction or Magnetic Flux Density (B)**

Whenever a piece of magnetic substance is placed in an external magnetising field, the substance becomes magnetised. If  $\mathbf{B}_0$  is the magnetic field in free space, then  $\mathbf{B} = \mu_r \mathbf{B}_0$ .

$\oint \mathbf{B} \cdot d\mathbf{S}$  is magnetic flux which is equal to  $\mu_0 m_{\text{inside}}$ , where  $m_{\text{inside}}$  is the net pole strength inside a close surface.

(ii) **Magnetic Permeability ( $\mu$ )** It is the degree or extent to which the magnetic lines of induction may pass through a given distance.

Magnetic permeability of free space  $\mu_0$  has a value of  $4\pi \times 10^{-7} \text{ TmA}^{-1}$ . However, for a material substance, absolute permeability ( $\mu$ ) has a value, different than  $\mu_0$ .

For any magnetic substance,  $\frac{\mu}{\mu_0} = \frac{\mathbf{B}}{\mathbf{B}_0} = \mu_r = \text{relative}$

magnetic permeability of that substance. **Relative magnetic permeability**  $\mu_r$  is a unitless and dimensionless term.

(iii) **Intensity of Magnetisation (I)** Intensity of magnetisation of a substance is defined as the magnetic moment induced in the substance per unit volume, when placed in the magnetising field.

Thus, 
$$\mathbf{I} = \frac{\mathbf{M}}{V}$$

It is a vector quantity and its SI unit is  $\text{Am}^{-1}$ .

(iv) **Intensity of Magnetising Field or Magnetic Intensity (H)**

It is a measure of the capability of external magnetising field to magnetise the given substance and is mathematically defined as

$$\mathbf{H} = \frac{\mathbf{B}_0}{\mu_0} \text{ or } \mathbf{H} = \frac{\mathbf{B}}{\mu} \text{ or } \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

Magnetic intensity  $\mathbf{H}$  is a vector quantity and its SI unit is  $\text{Am}^{-1}$ .

(v) **Magnetic Susceptibility ( $\chi_m$ )** Magnetic susceptibility of a substance is the ratio of the intensity of magnetisation  $I$  induced in the substance to the magnetic intensity  $H$ .

Thus,  $\chi_m = \frac{I}{H}$ . It is a scalar quantity and it has no units or dimensions.

**Relation between  $\mu_r$  and  $\chi_m$**  we have,  $B = \mu_0(I + H)$

$$\text{or } B = \mu_0 H \left( \frac{I}{H} + 1 \right) \text{ or } B = B_0 (\chi_m + 1) \text{ or } \frac{B}{B_0} = \chi_m + 1$$

But  $\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r = \text{relative permeability}$

$$\therefore \mu_r = \chi_m + 1$$

## Magnetic Materials

According to behaviour of magnetic substances, they are classified into three cases:

### Diamagnetic Materials

These are materials which show a very small decrease in magnetic flux, when placed in a strong magnetising field. Hydrogen, water, copper, zinc, antimony, bismuth, etc., are examples of diamagnetic materials.

- In a diamagnetic material, the net magnetic moment (sum of that due to orbital motion and spin motion of electrons) of an atom is zero. The external magnetic field  $\mathbf{B}$  distorts the electron orbit and thus induces a small magnetic moment in the opposite direction.
- Diamagnetic materials are feebly repelled in an external magnetic field and thus have a tendency to shift from the stronger to weaker regions of the magnetic field.
- The relative permeability of any diamagnetic substance is slightly less than 1 (i.e.  $\mu_r < 1$ ) and susceptibility has a small negative value.
- Diamagnetism is an intrinsic property and does not vary with magnetic field  $\mathbf{B}$  or temperature.

### Paramagnetic Materials

These are the materials which show a small increase in the magnetic flux when placed in a magnetising field. Oxygen, air, platinum, aluminium, etc., are examples of paramagnetic materials.

- In a paramagnetic material, the net magnetic moment of every atom is non-zero.
- Paramagnetic materials are feebly attracted in an external magnetic field and thus, have a tendency to shift from the weaker to the stronger regions of magnetic field.
- The relative permeability  $\mu_r$  of a paramagnetic material is slightly greater than one ( $\mu_r > 1$ ). Magnetic susceptibility  $\chi_m$  of paramagnetic materials is positive.
- Paramagnetism is temperature dependent. According to the **Curie's law**, the magnetic susceptibility of a paramagnetic substance is inversely proportional to its temperature  $T$ .

Mathematically,  $\chi_m = \frac{C}{T}$ , where  $C$  is the **Curie constant**.

### Ferromagnetic Materials

These are the materials which are strongly attracted by a magnetic field and can themselves be magnetised even in a weak magnetising field. Iron, steel, nickel and cobalt are ferromagnetic.

- These materials show a large increase in the magnetic flux, when placed in a magnetic field. Thus, for them  $\mu_r \gg 1$ . Accordingly,  $\chi_m$  is positive and large.
- Ferromagnetic materials exhibit all properties exhibited by paramagnetic substances and by a much larger measure.

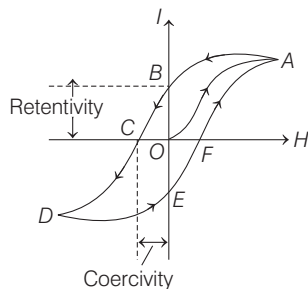
- Magnetic susceptibility of ferromagnetic materials decreases steadily with a rise in temperature. Above a certain temperature  $T_c$  (known as **Curie temperature**), the substance loses its ferromagnetic character and begins to behave as a paramagnetic substance.
- Above the Curie temperature  $T_c$ , the magnetic susceptibility of a ferromagnetic material varies as

$$\chi_m \propto \frac{1}{(T - T_c)} \quad \text{or} \quad \chi_m = \frac{C}{(T - T_c)}$$

where,  $C$  is a constant. It is known as the Curie-Weiss law.

## Hysteresis Curve

The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation and demagnetisation of a ferromagnetic material is called **hysteresis**.

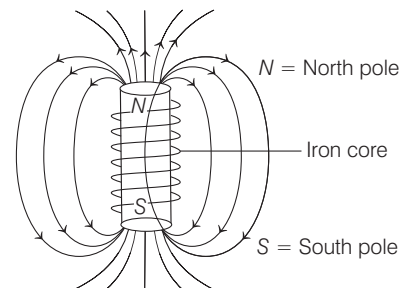


The whole graph  $ABCDEF$  is a closed loop and known as **hysteresis loop**.

## Electromagnet

Electromagnets are usually in the form of iron core solenoids. The ferromagnetic property of the iron core causes the internal magnetic domains of the iron to line up with the smaller driving magnetic field produced by the current in the solenoid.

The effect is the multiplication of the magnetic field by factors of ten to eleven thousands. The solenoid field relationship is  $B = k \mu_0 n I$ , where  $\mu = k \mu_0$  and  $k$  is the relative permeability of the iron, the figure shows the magnetic effect of the iron core.



## Permanent Magnet

Substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets. Permanent magnets can be made in a variety of ways.

An efficient way to make a permanent magnet is to place a ferromagnetic rod in a solenoid and pass a current. The magnetic field of the solenoid magnetise the rod.

### DAY PRACTICE SESSION 1

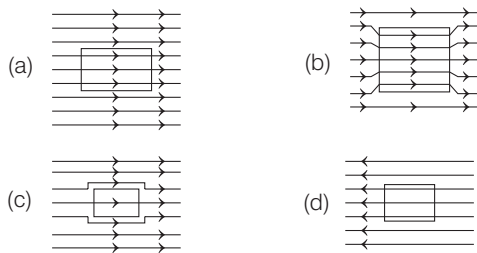
## FOUNDATION QUESTIONS EXERCISE

- Which of the following is most suitable as the core of transformers?
  - Steel
  - Alnico
  - Soft iron
  - None of these
- What is the angle of dip at a place, where horizontal component of earth's magnetic field is equal to the vertical component?
  - $0^\circ$
  - $30^\circ$
  - $45^\circ$
  - $90^\circ$
- In a certain place, the horizontal component of magnetic field is  $\frac{1}{\sqrt{3}}$  times the vertical component. The angle of dip at this place is
  - zero
  - $\pi/3$
  - $\pi/2$
  - $\pi/6$
- At a certain place, the angle of dip is  $60^\circ$  and the horizontal component of earth's magnetic field is  $0.4 \times 10^{-4}$  T. The earth's total magnetic field is
  - 0.02 mT
  - 0.04 mT
  - 0.08 mT
  - 0.16 mT
- At a place, the value of  $B_H$  and  $B_V$  are  $0.4 \times 10^{-4}$  T and  $0.3 \times 10^{-4}$  T. The angle of dip is
  - $\tan^{-1}\left(\frac{1}{2}\right)$
  - $\tan^{-1}(0.75)$
  - $\tan^{-1}\left(\frac{1}{3}\right)$
  - $\tan^{-1}(0.09)$

**6** The correct value of the dip angle at a place is  $45^\circ$ . The dip circle is rotated through  $45^\circ$  out of the magnetic meridian. The apparent angle of dip will be

- (a)  $\tan^{-1}(\sqrt{2})$  (b)  $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$  (c)  $\tan^{-1}(2)$  (d)  $\tan^{-1}\left(\frac{1}{2}\right)$

**7** A uniform magnetic field parallel to the plane of paper, existed in space initially directed from left to right. When a bar of soft iron is placed in the field parallel to it, the lines of force passing through it will be represented by figure



**8** If a diamagnetic substance is brought near the North or the South pole of a bar magnet, it is → CBSE AIPMT 2009

- (a) repelled by both the poles  
 (b) repelled by the North pole and attracted by the South pole  
 (c) attracted by the North pole and repelled by the South pole  
 (d) attracted by both the poles

**9** Nickel shows ferromagnetic property at room temperature. If the temperature is increased beyond Curie temperature, then it will show

- (a) paramagnetism (b) anti-ferromagnetism  
 (c) no magnetic property (d) diamagnetism

**10** Above Curie temperature,

- (a) a ferromagnetic substance becomes paramagnetic  
 (b) a paramagnetic substance becomes diamagnetic  
 (c) a diamagnetic substance becomes paramagnetic  
 (d) a paramagnetic substance becomes ferromagnetic

**11** The susceptibility of the diamagnetic substance is

- (a) very large (b) positive and small  
 (c) zero (d) negative

**12** Which one of the following is not made of soft iron?

- (a) Electromagnet → AIIMS 2011  
 (b) Core of transformer  
 (c) Core of dynamo  
 (d) Magnet of loudspeaker

**13** The magnetic susceptibility is negative for → NEET 2016

- (a) paramagnetic material only  
 (b) ferromagnetic material only  
 (c) paramagnetic and ferromagnetic materials  
 (d) diamagnetic material only

**14** The permeability of the paramagnetic substance is

- (a) very large (b) small, but more than 1  
 (c) less than 1 (d) negative

**15** What is the relation between permeability  $\mu$ , permeability of free space  $\mu_0$  and susceptibility  $\chi_m$ ?

- (a)  $\mu = \mu_0 (1 + \chi_m)$   
 (b)  $\mu = \mu_0 (1 - \chi_m)$   
 (c)  $\mu_0 = \mu (1 + \chi_m)$   
 (d)  $\mu_0 = \mu (1 - \chi_m)$

**16** Hysteresis cycle for the material of permanent magnet should be

- (a) high and high (b) high and thin  
 (c) low and high (d) low and thin

**17** Electromagnets are made of soft iron, because soft iron has → CBSE AIPMT 2010

- (a) low retentivity and high coercive force  
 (b) high retentivity and high coercive force  
 (c) low retentivity and low coercive force  
 (d) high retentivity and low coercive force

**18** There are four light weight rod samples *A*, *B*, *C* and *D* separately suspended by thread. A bar magnet is slowly brought near each sample and the following observations are noted

- (i) *A* is feebly repelled  
 (ii) *B* is feebly attracted  
 (iii) *C* is strongly attracted  
 (iv) *D* remains unaffected

Which one of the following is true? → CBSE AIPMT 2011

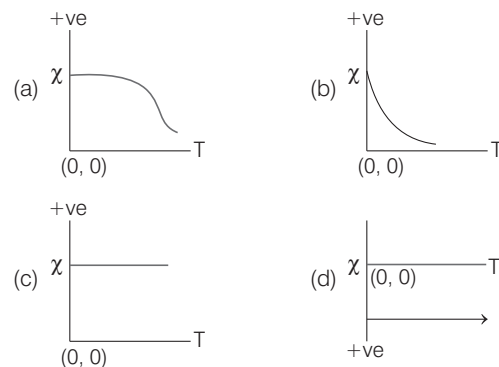
- (a) *C* is of a diamagnetic material  
 (b) *D* is of a ferromagnetic material  
 (c) *A* is of a non-magnetic material  
 (d) *B* is of a paramagnetic material

**19** The magnetic susceptibility of a material of a rod is 499.

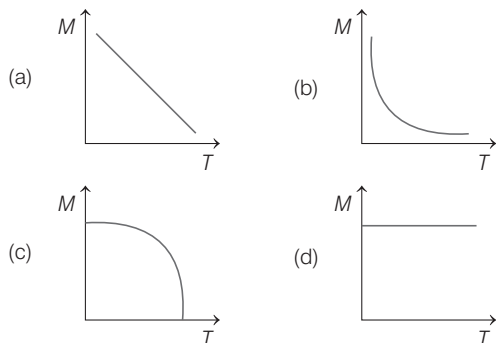
Permeability of vacuum is  $4\pi \times 10^{-7} \text{ Hm}^{-1}$ . Absolute permeability of the material of the rod (in  $\text{Hm}^{-1}$ ) is

- (a)  $\pi \times 10^{-4}$  (b)  $2\pi \times 10^{-4}$   
 (c)  $3\pi \times 10^{-4}$  (d)  $4\pi \times 10^{-4}$

**20** The variation of magnetic susceptibility ( $\chi$ ) with absolute temperature *T* for a ferromagnetic is given in figure, by



21 A curve between magnetic moment and temperature of magnet is



22 A tangent galvanometer of reduction factor 1A is placed with the plane of its coil parallel to the magnetic meridian, when a current of 1A is passed through it, the deflection produced is

- (a)  $45^\circ$  (b) zero  
(c)  $30^\circ$  (d)  $60^\circ$

23 How much work is done to rotate a magnet of dipole moment  $\rho_m$  in a magnetic field  $B$  through  $180^\circ$ ?

- (a)  $\rho_m \frac{B}{2}$  (b)  $\rho_m B$   
(c)  $3\rho_m \frac{B}{2}$  (d)  $2\rho_m B$

24 A current loop in a magnetic field → NEET 2013

- (a) experiences a torque whether the field is uniform or non-uniform in all orientations  
(b) can be in equilibrium in one orientation  
(c) can be equilibrium in two orientations, both the equilibrium states are unstable  
(d) can be in equilibrium in two orientations, one stable, while the other is unstable

25 A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It

→ CBSE AIPMT 2012

- (a) will become rigid showing no movement  
(b) will stay in any position  
(c) will stay in North-South direction only  
(d) will stay in East-West direction only

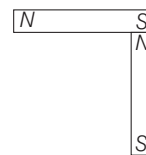
26 Two bar magnets of the same mass, same length and breadth, but having magnetic moments  $M$  and  $2M$  are joined together pole to pole and suspended by a string. The time period of assembly in a magnetic field of strength  $B$  is 3s. If now the polarity of one of the magnet is reversed and the combination is again made to oscillate in the same field, then the time of oscillation is

- (a)  $\sqrt{3}$  s (b)  $3\sqrt{3}$  s (c) 3 s (d) 6 s

27 The period of oscillation of a magnet in a vibration magnetometer is 8s. The period of oscillation of magnet whose magnetic moment is four times that of the first magnet is

- (a) 1 s (b) 4 s (c) 8 s (d) 16 s

28 Two identical bar magnets each of dipole moment  $\rho_m$  and length  $l$  are perpendicular to each other as shown in the figure. The dipole moment of the combination is



- (a)  $2\rho_m$  (b)  $\sqrt{2}\rho_m$   
(c)  $\frac{\rho_m}{\sqrt{2}}$  (d)  $\frac{\rho_m}{2}$

29 A thin bar magnet vibrates in the horizontal plane with a period of 8s. The magnet is cut into two halves perpendicular to magnetic axis. Then, the period of vibration of each half is approximately

- (a) 4 s (b) 2 s  
(c) 1 s (d) 0.5 s

30 If the magnet is suspended at an angle  $30^\circ$  to the magnetic meridian, the dip needle makes an angle of  $60^\circ$  with the horizontal. What is the true dip?

- (a)  $\tan^{-1}(3)$  (b)  $\tan^{-1}(3/2)$   
(c)  $\tan^{-1}(2/3)$  (d)  $\tan^{-1}(1/3)$

31 A coil in the shape of an equilateral triangle of side 0.02 m is suspended from its vertex, such that it is hanging in a vertical plane between the pole pieces of permanent magnet producing a uniform field of  $5 \times 10^{-2}$  T. If a current of 0.1 A is passed through the coil, what is the torque acting on it?

- (a)  $5\sqrt{3} \times 10^{-7}$  N-m (b)  $5\sqrt{3} \times 10^{-10}$  N-m  
(c)  $\frac{\sqrt{3}}{5} \times 10^{-7}$  N-m (d) None of these

32 A North pole of 40 A-m is placed 20 cm apart from a South pole of 80 A-m. Calculate the distance of a point from the South pole on the line joining, the two poles where the resultant field due to these poles is zero

- (a) 8.2 cm towards North pole  
(b) 8.2 cm away from North pole  
(c) 48.2 cm towards North pole  
(d) 48.2 cm away from North pole

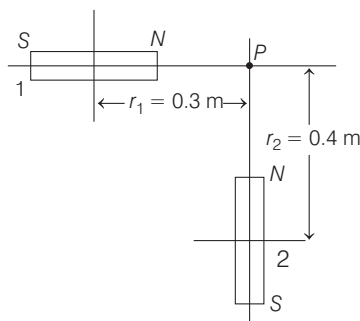
33 A circular current loop of magnetic moment  $M$  is in an arbitrary orientation in an external magnetic field  $B$ . The work done to rotate the loop by  $30^\circ$  about an axis perpendicular to its plane, is

- (a)  $MB$  (b)  $\sqrt{3} \frac{MB}{2}$   
(c)  $\frac{MB}{2}$  (d) zero

## DAY PRACTICE SESSION 2

# PROGRESSIVE QUESTIONS EXERCISE

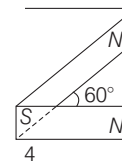
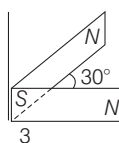
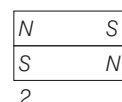
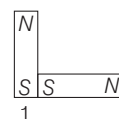
- 1** At the magnetic pole of the earth, the value of angle of dip is  
 (a)  $0^\circ$  (b)  $30^\circ$   
 (c)  $45^\circ$  (d)  $90^\circ$
- 2** A small magnet of dipole moment  $M$  is kept on the arm of a deflection magnetometer set in  $\tan A$  position at a distance of 0.2 m. If the deflection is  $60^\circ$ , the value of  $M$  is (Take,  $B_H = 0.4 \times 10^{-4} \text{ T}$ )  
 (a)  $2.77 \text{ A-m}^2$  (b) zero  
 (c)  $10.82 \text{ A-m}^2$  (d) None of these
- 3** Two short magnets of magnetic moment  $2 \text{ A-m}^2$  and  $5 \text{ A-m}^2$  are placed along two lines drawn at right angle to each other on the sheet of paper as shown in the figure. What is magnetic field at the point of intersection of their axis?



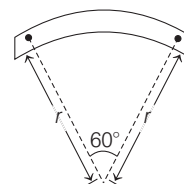
- (a) 0 (b)  $215 \times 10^{-5} \text{ T}$   
 (c)  $2.15 \times 10^{-5} \text{ T}$  (d)  $0.215 \text{ T}$
- 4** Two tangent galvanometer having coils of the same radius are connected in series. A current flowing in them, produces deflection of  $60^\circ$  and  $45^\circ$ , respectively. The ratio of the number of turns in the coils is  
 (a)  $\frac{4}{3}$  (b)  $\frac{\sqrt{3} + 1}{1}$  (c)  $\frac{\sqrt{3}}{1}$  (d)  $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$
- 5** In  $\tan A$  position, two short bar magnets of moments in the ratio 1:1.7321 are placed at the same distance separately. If the deflection produced for the first magnet is  $30^\circ$ , then the deflection produced for the second magnet will be  
 (a)  $60^\circ$  (b)  $30^\circ$   
 (c)  $45^\circ$  (d)  $42^\circ$
- 6** A vibration magnetometer consists of two identical bar magnets placed one over the other, such that they are perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is  $2^{5/4} \text{ s}$ . One of

the magnets is removed and if the other magnet oscillates in the same field, then the time period (in second) is  
 (a)  $2^{1/4}$  (b)  $2^{1/2}$  (c) 2 (d)  $2^{3/4}$

- 7** A vibration magnetometer placed in magnetic meridian has a small bar magnet. The magnet executes oscillations with the time period of 2 s in earth's horizontal magnetic field of  $24 \mu\text{T}$ . When a horizontal field of  $18 \mu\text{T}$  is produced opposite to the earth's field by placing a current carrying wire, the new time period of magnet will be  
 (a) 1 s (b) 2 s (c) 3 s (d) 4 s
- 8** Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment  $m$ . Which configuration has highest net magnetic dipole moment? → CBSE AIPMT 2014



- (a) 1 (b) 2 (c) 3 (d) 4
- 9** A bar magnet of length  $l$  and magnetic dipole moment  $M$  is bent in the form of an arc as shown in figure. The new magnetic dipole moment will be → NEET 2013



- (a)  $M$  (b)  $\frac{3}{\pi} M$  (c)  $\frac{2}{\pi} M$  (d)  $\frac{M}{2}$
- 10** A thin magnetic needle vibrates in the horizontal plane with a period of 4s. The needle is cut into two halves by a plane normal to magnetic axis of the needle. Then, the period of vibration of each half needle is approximately  
 (a) 4 s (b) 2 s  
 (c) 8 s (d) 1 s

**11** A thin diamagnetic rod is placed vertically between the poles of an electromagnet. When the current in the electromagnet is switched ON, then the diamagnetic rod is pushed up, out of the horizontal magnetic field. Hence, the rod gains gravitational potential energy. The work required to do this comes from → NEET 2018

- (a) the lattice structure of the material of the rod  
 (b) the magnetic field  
 (c) the current source  
 (d) the induced electric field due to the changing magnetic field

**12** Two magnets placed one above the other oscillate with a period of 17 s. If one of them is reversed the time period becomes 8 s. The ratio of their magnetic moment is nearest to

- (a)  $\frac{8}{17}$  (b)  $\frac{9}{14}$  (c)  $\frac{1}{4}$  (d)  $\frac{1}{2}$

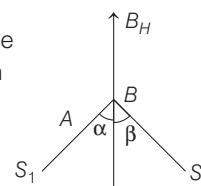
**13** A bar magnet is hung by a thin cotton thread in a uniform horizontal magnetic field and is in equilibrium state. The energy required to rotate it by  $60^\circ$  is  $W$ . Now the torque required to keep the magnet in this new position is → NEET 2016

- (a)  $\frac{W}{\sqrt{3}}$  (b)  $\sqrt{3}W$  (c)  $\frac{\sqrt{3}W}{2}$  (d)  $\frac{2W}{\sqrt{3}}$

**14** A bar magnet having a magnetic moment of  $2 \times 10^4 \text{ JT}^{-1}$  is free to rotate in a horizontal plane. A horizontal magnetic field  $B = 6 \times 10^{-4} \text{ T}$  exists in the space. The work done in taking the magnet slowly from a direction parallel to the field to a direction  $60^\circ$  from the field is → CBSE AIPMT 2009

- (a) 0.6 J (b) 12 J (c) 6 J (d) 2 J

**15** Two small magnets  $A$  and  $B$  of dipole moments  $M_0$  and  $2M_0$  respectively, are fixed perpendicular to each other with their North poles in contact. The combination is placed on a floating body, so as to move freely in earth's magnetic field, the value of  $\alpha$  is



- (a)  $\tan^{-1}(2)$  (b)  $\sin^{-1}\left(\frac{1}{2}\right)$   
 (c)  $\cos^{-1}\left(\frac{1}{2}\right)$  (d)  $20^\circ$

**16** A current carrying circular loop of radius  $R$  is placed in the  $XY$ -plane with centre at the origin. Half of the loop with  $x > 0$  is now bent, so that it now lies in the  $YZ$ -plane.

- (a) The magnitude of magnetic moment now diminishes  
 (b) The magnetic moment does not change  
 (c) The magnitude of  $\mathbf{B}$  at  $(0, 0, z)$ ,  $z \gg R$  increases  
 (d) The magnitude of  $\mathbf{B}$  at  $(0, 0, z)$ ,  $z \gg R$  unchanged

**17** A bar magnet is suspended with unspun thread. It lies in the magnetic meridian. The upper end of the thread is turned through  $150^\circ$ . It turns the magnet by  $30^\circ$  with the magnetic meridian. Through what angle should the upper end of the thread be turned to rotate the magnet through  $90^\circ$ ?

- (a)  $360^\circ$  (b)  $330^\circ$  (c)  $300^\circ$  (d)  $270^\circ$

**18** If  $\theta_1$  and  $\theta_2$  be the apparent angles of dip observed in two vertical planes at right angles to each other, then the true angle of dip  $\theta$  is given by → NEET 2017

- (a)  $\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$  (b)  $\tan^2 \theta = \tan^2 \theta_1 + \tan^2 \theta_2$   
 (c)  $\cot^2 \theta = \cot^2 \theta_1 - \cot^2 \theta_2$  (d)  $\tan^2 \theta = \tan^2 \theta_1 - \tan^2 \theta_2$

## ANSWERS

SESSION 1	1 (c)	2 (c)	3 (b)	4 (c)	5 (b)	6 (a)	7 (b)	8 (a)	9 (a)	10 (a)
	11 (d)	12 (d)	13 (d)	14 (b)	15 (a)	16 (a)	17 (d)	18 (d)	19 (b)	20 (a)
	21 (c)	22 (a)	23 (d)	24 (d)	25 (c)	26 (b)	27 (b)	28 (b)	29 (a)	30 (b)
	31 (a)	32 (c)	33 (d)							
SESSION 2	1 (d)	2 (a)	3 (c)	4 (c)	5 (c)	6 (c)	7 (b)	8 (c)	9 (b)	10 (b)
	11 (c)	12 (b)	13 (b)	14 (c)	15 (a)	16 (a)	17 (b)	18 (a)		



# Hints and Explanations

**1** The core of the transformer should have high permeability and low coercivity. The soft iron have high permeability and lens coercivity. So, the core of transformers are made of soft iron.

**2** The dip angle is  $45^\circ$  at a place, where horizontal component of earth's magnetic field is equal to the vertical component.

**3** From  $\tan \delta = \frac{V}{H} = \frac{V}{V/\sqrt{3}} = \sqrt{3}$   
 $\therefore \delta = \pi/3$

**4**  $B_H = B_0 \cos \delta$ . Hence,  $B_0 = \frac{B_H}{\cos \delta}$

Here,  $\delta = 60^\circ$  and  $B_H = 0.4 \times 10^{-4}$  T

$$\Rightarrow B_0 = \frac{0.4 \times 10^{-4}}{\cos 60^\circ} = 0.8 \times 10^{-4}$$

$$\Rightarrow B_0 = 0.08 \text{ mT}$$

**5**  $\therefore \tan \delta = \frac{B_V}{B_H}$

$$\tan \delta = \frac{0.3 \times 10^{-4}}{0.4 \times 10^{-4}} = \frac{3}{4}$$

$$\Rightarrow \delta = \tan^{-1} \left( \frac{3}{4} \right) = \tan^{-1} (0.75)$$

**6**  $\tan \delta = \frac{V}{H} = 45^\circ$ . Hence,  $V = H$

When rotated through  $45^\circ$  out of magnetic meridian, the vertical component remains unchanged, but the horizontal component changes to  $H \cos 45^\circ = H/\sqrt{2}$ .

$$\text{Hence, } \tan \delta' = \frac{V}{H/\sqrt{2}} = \sqrt{2}$$

$$\Rightarrow \delta' = \tan^{-1} \sqrt{2}$$

**7** Because of large permeability of soft iron magnetic lines of force prefer to pass through it. Concentration of lines in soft iron bar increases as shown in figure (b).

**8** Diamagnetic substances are weakly magnetised in a direction opposite to that of applied magnetic field. These are repelled in an external magnetic field, i.e. have a tendency to move from high to low field region, i.e. it is repelled by both North and South poles of a bar magnet.

**9** If the temperature of a ferromagnetic material is raised above a certain critical value, called the Curie temperature, the exchange coupling ceases to be effective. Most such materials, then

become simply paramagnetic, i.e. the dipoles still tend to align with an external field, but much more weakly and thermal agitation can now more easily disrupt the alignment.

**10** Ferromagnetism decreases with rise in temperature. So, the temperature above which a ferromagnetic substance becomes paramagnetic is called the Curie temperature of the substance.

**11** The susceptibility of the diamagnetic substance is less than zero or negative.

**12** Temporary magnet is made from soft iron, while magnet of loudspeaker is made from permanent magnet.

**13** As we know the relation between the magnetic permeability and susceptibility of material, i.e.

$$\mu_r = 1 + \chi_m \quad \dots(i)$$

$\therefore$  For diamagnetic substances,  $\mu_r < 1$   
 So, according to Eq. (i), the magnetic susceptibility ( $\chi_m$ ) of diamagnetic substance will be negative.

**14** The permeability of the paramagnetic substance is small, but more than 1.

**15** The relation between permeability  $\mu$ , permeability of free space  $\mu_0$  and susceptibility  $\chi_m$  is,  $\mu = \mu_0 (1 + \chi_m)$

**16** The material of permanent magnet should have high retentivity as well as high coercivity.

**17** The material suitable for making electromagnets should have high retentivity and low coercivity.

**18** Paramagnetic material will be feebly attracted, diamagnetic material will be feebly repelled and ferromagnetic material will be strongly attracted.

**19** Magnetic susceptibility,  $\chi = \mu_r - 1$

$$\text{or } \chi = \frac{\mu}{\mu_0} - 1$$

$$\text{or } 499 = \frac{\mu}{4\pi \times 10^{-7}} - 1$$

$$\text{or } \mu = 500 \times 4\pi \times 10^{-7} \\ = 2\pi \times 10^{-4} \text{ Hm}^{-1}$$

**20** As temperature of a ferromagnetic material is raised, its susceptibility  $\chi$  remains constant first and then decreases as shown in figure (a).

**21** With rise in temperature, the magnetism of magnet falls and at critical temperature, it becomes zero.

$$\begin{aligned} \mathbf{22} \quad I &= k \tan \theta \\ \Rightarrow \tan \theta &= \frac{I}{k} = \frac{1}{1} = 1 \\ \therefore \theta &= 45^\circ \end{aligned}$$

$$\begin{aligned} \mathbf{23} \quad W &= \int_0^\theta \tau \cdot d\theta = \int_0^\theta p_m B_H \sin \theta d\theta \\ &= p_m B_H [-\cos \theta]_0^\theta \\ &= p_m B_H [1 - \cos \theta] \end{aligned}$$

When  $\theta = \pi$ , then  $\cos \theta = -1$

$$T' = \left[ \frac{2}{8} \right]^{1/2} T = \frac{T}{2} = \left( \frac{8}{2} \right) \text{ s} = 4 \text{ s}$$

$$\text{Hence, } W = 2 p_m B_H = 2 p_m B$$

**24** For like parallel magnetic field is stable and for anti-parallel is unstable.

**25** It will stay in any position at geomagnetic North and South poles.

$$\mathbf{26} \quad \therefore \frac{T_2}{T_1} = \frac{(3M)^{1/2}}{(M)^{1/2}} = \frac{\sqrt{3}}{1}$$

Given,  $T_1 = 3 \text{ s}$ , then we get  $T_2 = 3\sqrt{3} \text{ s}$

**27**  $T = 2\pi [I/p_m B]^{1/2}$ . So, when  $p_m$  becomes 4 times,  $T$  becomes half.

$$\therefore T = \frac{8}{2} = 4 \text{ s}$$

**28** Distance between the free North and South poles is  $\sqrt{2} L$ . Hence, dipole moment of the combination is  $\sqrt{2} p_m$ .

**29**  $T = 2\pi [I/p_m B]^{1/2}$ .  $p_m = q_m l$ . On cutting  $l$  is reduced to half, so  $p_m$  also becomes half. For a thin bar, the moment of inertia,  $I = \frac{M}{12} [l^2 + b^2] \approx \frac{M}{12} l^2$ . Because for thin bar,  $b$  is negligible. On cutting  $l$  becomes half. Also,  $M$  becomes half. So, the moment of inertia becomes one-eighth. Hence, new time period is

$$T' = \left( \frac{2}{8} \right)^{1/2} T = \frac{T}{2} = \left( \frac{8}{2} \right) \text{ s} = 4 \text{ s}$$

$$\mathbf{30} \quad \tan \delta = \frac{V}{H} \quad \text{and} \quad \tan \delta' = \frac{V}{H \cos \theta}$$

$$\text{or } \tan \delta' = \frac{\tan \delta}{\cos \theta}$$

$$\begin{aligned} \Rightarrow \tan \delta &= \tan \delta' \cos \theta \\ &= \tan (60^\circ) \cos (30^\circ) \\ &= \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2} \end{aligned}$$

$$\Rightarrow \delta = \tan^{-1} \left( \frac{3}{2} \right)$$



**31** Torque,  $\tau = LAB \sin\theta$ ,  $I = 0.1$  A,  $\theta = 90^\circ$

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{or } A = \frac{1}{2} a \times \frac{a\sqrt{3}}{2}$$

$$= \frac{\sqrt{3} a^2}{4} = \frac{\sqrt{3} \times (0.02)^2}{4}$$

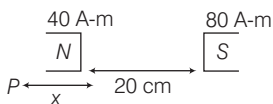
$$= \sqrt{3} \times 10^{-4} \text{ m}^2$$

$$\tau = 0.1 \times \sqrt{3} \times 10^{-4} \times 5 \times 10^{-2} \sin 90^\circ$$

$$= 5\sqrt{3} \times 10^{-7} \text{ N-m}$$

**32** The situation is shown in figure.

Let at point  $P$ , magnetic field is zero.

$$\therefore \frac{\mu_0 m_1}{4\pi r_1^2} = \frac{\mu_0 m_2}{4\pi r_2^2}$$


$$\frac{40 \text{ A-m}}{x^2} = \frac{80 \text{ A-m}}{(20+x)^2}$$

$$(20+x)^2 = 2x^2$$

$$\Rightarrow 20+x = \sqrt{2}x$$

$$\therefore x = \frac{20}{\sqrt{2}-1}$$

$$= 48.2 \text{ cm (towards North pole)}$$

**33** Let magnetic moment  $\mathbf{M}$  of current loop be making an angle  $\alpha$  with the direction of  $\mathbf{B}$ . When a current circular loop is rotated in a magnetic field by  $30^\circ$  about an axis perpendicular to its plane, there is no change in the angle  $\alpha$  between magnetic moment  $\mathbf{M}$  and magnetic field  $\mathbf{B}$ . Therefore,  $\theta_1 = \alpha$ ,  $\theta_2 = \alpha$ .

$$\text{Work done, } W = MB(\cos\theta_1 - \cos\theta_2)$$

$$= MB(\cos\alpha - \cos\alpha) = 0$$

## SESSION 2

**1** At magnetic poles, the angle of dip is  $90^\circ$ .

**2**  $B = B_H \tan\theta$

$$\frac{\mu_0 2M}{4\pi r^3} = 0.4 \times 10^{-4} \tan 60^\circ$$

$$\frac{\mu_0 M}{2\pi (0.2)^3} = 0.4 \times 10^{-4} \tan 60^\circ$$

$$\Rightarrow M = \frac{0.4 \times 2\pi \times (0.2)^3 \times (10^{-4}) \times \sqrt{3}}{\mu_0}$$

$$= 2.77 \text{ A-m}^2$$

**3** Magnetic field due to magnet 1,

$$B_1 = \frac{\mu_0 2M_1}{4\pi r_1^3}$$

$$= \frac{10^{-7} \times 2 \times 2}{(0.3)^3}$$

$$B_1 = 1.48 \times 10^{-5} \text{ T}$$

Magnetic field due to magnet 2,

$$B_2 = \frac{\mu_0 2M_2}{4\pi r_2^3} = \frac{10^{-7} \times 2 \times 5}{(0.4)^3}$$

$$= 1.56 \times 10^{-5} \text{ T}$$

Net field at  $P$ ,

$$B = \sqrt{B_1^2 + B_2^2} = 2.15 \times 10^{-5} \text{ T}$$

**4** For tangent galvanometer,

$$\frac{\mu_0 nI}{2r} = H \tan\theta$$

$$n \propto H \tan\theta$$

$$\therefore \frac{n_1}{n_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{\tan 60^\circ}{\tan 45^\circ} = \frac{\sqrt{3}}{1}$$

**5**  $B = B_E \tan\theta$  or  $\frac{\mu_0 2M}{4\pi r^2} = B_E \tan\theta$

$$\text{or } M \propto \tan\theta$$

$$\therefore \frac{M_1}{M_2} = \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1}{1.7321}$$

$$\text{or } \tan\theta = 1 \text{ or } \theta = 45^\circ$$

**6**  $T = 2\pi \sqrt{\frac{I}{MB}} \propto \sqrt{\frac{I}{M}}$

$$\therefore \frac{T'}{T} = \sqrt{\frac{I' M}{I M'}} = \sqrt{\frac{I \sqrt{2} M}{2I M}} = \frac{1}{2^{1/4}}$$

$$\text{or } T' = \frac{2^{5/4}}{2^{1/4}} = 2 \text{ s}$$

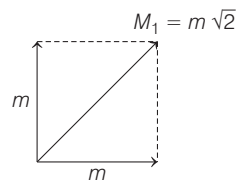
**7** Time period in vibration magnetometer,

$$T = 2\pi \sqrt{\frac{I}{M \times B_H}} \Rightarrow T \propto \frac{1}{\sqrt{B_H}}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\sqrt{(B_H)_2}}{\sqrt{(B_H)_1}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{18}{24}}$$

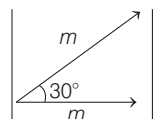
$$T = 2.3 \text{ s} \approx 2 \text{ s}$$

**8** (1)



(2)  $\frac{m}{m} \Rightarrow M_2 = 0$

(3)

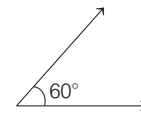


$$M_3 = m \sqrt{(1 + \cos 30^\circ) 2}$$

$$= m \sqrt{\left(1 + \frac{\sqrt{3}}{2}\right) 2}$$

$$= m \sqrt{2 + \sqrt{3}}$$

(4)



$$M_4 = 2m \cos 30^\circ = m\sqrt{3}$$

**9** The magnetic moment,

$$M = ml$$

$$\text{From figure, } l = \frac{\pi}{3} \times r \text{ or } r = \frac{3l}{\pi}$$

$\therefore$  New magnetic moment,

$$M' = m \times r = m \times \frac{3l}{\pi}$$

$$= \frac{3}{\pi} \cdot ml = \frac{3M}{\pi}$$

**10** The moment of inertia of needle, when it cuts into two halves by a plane normal to magnetic axis of the needle.

$$I' = \frac{1}{12} \left(\frac{m}{2}\right) \left(\frac{l}{2}\right)^2 \text{ or } I' = \frac{I}{8}$$

$$\text{Also, } M' = \frac{M}{2}$$

$$T' = 2\pi \sqrt{\frac{I'}{M' B}}$$

The time period of vibration

$$= 2\pi \sqrt{\frac{I \times 2}{8 \times MB}} = \frac{1}{2} T = \frac{1}{2} 4 \text{ s} = 2 \text{ s}$$

**11** As the source of current is switched ON, a magnetic field sets up in between the poles of the electromagnet.

As we know that a diamagnetic substance when placed in a magnetic field acquires a feeble magnetism opposite to the direction of magnetic field.

Also, in the presences of the field (non-uniform), these substances are attracted towards the weaker field, i.e. they move from stronger to weaker magnetic field.

Due to these reasons, the rod is repelled by the field produced to the current source. Hence, it is pushed up, out of horizontal field and gains gravitational potential energy.

$$\mathbf{12} \quad T_1 = 2\pi \left[ \frac{I_1 + I_2}{(P_{m_1} - P_{m_2}) B} \right]^{1/2}$$

$$\text{and } T_2 = 2\pi \left[ \frac{I_1 + I_2}{(P_{m_1} + P_{m_2}) B} \right]^{1/2}$$

$$\text{Therefore, } \frac{T_1^2}{T_2^2} = \frac{P_{m_1} + P_{m_2}}{P_{m_1} - P_{m_2}}$$

This gives,

$$\frac{T_1^2 - T_2^2}{T_1^2 + T_2^2} = \frac{P_{m_2}}{P_{m_1}} = \frac{(17)^2 - (8)^2}{(17)^2 + (8)^2}$$

$$= \frac{225}{353} \approx \frac{9}{14}$$

**13** ∴ Work done in rotating the magnet,  
 $W = MB (\cos \theta_0 - \cos \theta)$   
 where,  $M$  = magnetic moment of  
 the magnet

and  $B$  = magnetic field.  
 $W = MB (\cos 0^\circ - \cos 60^\circ)$   
 $= MB \left(1 - \frac{1}{2}\right) = \frac{MB}{2}$

∴  $MB = 2W$  ... (i)

Torque on a magnet in this position is given by

$\tau = \mathbf{M} \times \mathbf{B}$   
 $= MB \cdot \sin \theta = 2W \cdot \sin 60^\circ$   
 [from Eq. (i)]

$= 2W \frac{\sqrt{3}}{2} = \sqrt{3}W$

**14** When magnetic dipole is rotated from initial position  $\theta_1$  to final position  $\theta_2$ , then work done

$= MB(\cos \theta_1 - \cos \theta_2)$

Given,  $\theta_1 = 0^\circ, \theta_2 = 60^\circ$

Magnetic moment,  $M = 2 \times 10^4$  J/T

Magnetic field,  $B = 6 \times 10^{-4}$  T

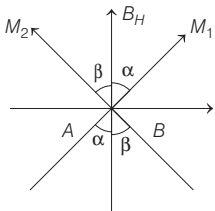
So,  $W = MB \left(1 - \frac{1}{2}\right)$

$\left[ \because \cos 0^\circ = 1 \right.$   
 $\left. \text{and } \cos 60^\circ = 1/2 \right]$

$= \frac{2 \times 10^4 \times 6 \times 10^{-4}}{2} = 6$  J

**15** For equilibrium,

$M_1 \sin \alpha = M_2 \sin \beta$   
 [∴  $\alpha + \beta = 90^\circ$ ]



$M_1 \sin \alpha = M_2 \sin(90 - \alpha)$   
 [∴  $\alpha + \beta = 90^\circ$ ]

$M_1 \sin \alpha = M_2 \cos \alpha$   
 $M_0 \sin \alpha = 2M_0 \cos \alpha$

⇒  $\tan \alpha = 2$   
 $\alpha = \tan^{-1}(2)$

**16** The magnetic moment,  $M = I \times \pi R^2$ . It acts perpendicular to the loop along z-direction. When half of the current loop is bent in YZ-plane, then magnetic moment due to half current loop in XY-plane,  $M_1 = I(\pi R^2/2)$  acting along z-direction.

Magnetic moment due to half current loop in YZ-plane,  $M_2 = I(\pi R^2/2)$  along x-direction.

Effective magnetic moment due to entire bent current loop,

$M' = \sqrt{M_1^2 + M_2^2}$   
 $= \sqrt{(I\pi R^2/2)^2 + (I\pi R^2/2)^2}$   
 $= \frac{I\pi R^2}{2} \sqrt{2} < M$

i.e. Magnetic moment diminishes.

**17** The twist in the thread is  $150^\circ - 30^\circ = 120^\circ$ .

Restoring torque,

$p_m B \sin 30^\circ = p_m \frac{B}{2}$

Hence,  $\frac{p_m B}{2} \propto 120^\circ$  ... (i)

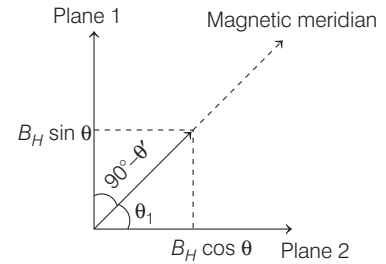
In the second case, let the thread be turned through angle  $\theta$ . Then, twist in the thread  $(\theta - 90^\circ)$ . And restoring torque is

$p_m B \sin 90^\circ = p_m B$ .

Therefore,  $p_m B \propto (\theta - 90^\circ)$  ... (ii)

From Eqs. (i) and (ii), we get  
 $\theta = 330^\circ$

**18** Let  $B_H$  and  $B_V$  be the horizontal and vertical component of earth's magnetic field  $\mathbf{B}$ .



$\tan \theta = \frac{B_V}{B_H} \Rightarrow \cot \theta = \frac{B_H}{B_V}$  ... (i)

Let planes 1 and 2 are mutually perpendicular planes making angle  $\theta$  and  $(90^\circ - \theta)$  with magnetic meridian. The vertical component of earth's magnetic field remain same in two plane, but effective horizontal components in the two planes is given by

$B_1 = B_H \cos \theta'$  ... (ii)

and  $B_2 = B_H \sin \theta'$  ... (iii)

Then,  $\tan \theta_1 = \frac{B_V}{B_1} = \frac{B_V}{B_H \cos \theta'}$

$\cot \theta_1 = \frac{B_H \cos \theta'}{B_V}$  ... (iv)

Similarly,  $\Rightarrow \tan \theta_2 = \frac{B_V}{B_2} = \frac{B_V}{B_H \sin \theta'}$

$\Rightarrow \cot \theta_2 = \frac{B_H \sin \theta'}{B_V}$  ... (v)

From Eqs. (iv) and Eq. (v), we get

$\Rightarrow \cot^2 \theta_1 + \cot^2 \theta_2$

$= \frac{B_H^2 \cos^2 \theta'}{B_V^2} + \frac{B_H^2 \sin^2 \theta'}{B_V^2}$

$\Rightarrow \cot^2 \theta_1 + \cot^2 \theta_2$

$= \frac{B_H^2}{B_V^2} (\cos^2 \theta' + \sin^2 \theta')$

$\Rightarrow \cot^2 \theta_1 + \cot^2 \theta_2 = \cot^2 \theta$